DAY THIRTY FIVE

Probability

Learning & Revision for the Day

- Sample Space and Event
- Probability of an Event
- Conditional Probability
- Multiplication Theorem on Probability
- Theorem of Total Probability
- Baye's Theorem
- Probability Distribution of a Random Variable
- Bernoulli Trials and Binomial Distribution

Sample Space and Event

The **sample space** is the set of all possible outcomes of an random experiment and it is generally denoted by S(e.g. tossing a coin, rolling a die, drawing a card from a pack of playing cards etc.).

An event is a subset of S. If a die is rolled, then $S = \{1, 2, 3, 4, 5, 6\}$ is the sample space and getting an odd number $A = \{1, 3, 5\}$ is an event.

Types of Events

Equally Likely Event

The given events are said to be **equally likely,** if none of them is expected to occur in preference to the other.

Mutually Exclusive/Disjoint

A set of events is said to be **mutually exclusive,** if occurrence of one of them prevents or denies the occurrence of any of the remaining events.

If a set of events $E_1, E_2, \ldots, E_n$ are mutually exclusive events, then $E_1 \cap E_2 \cap \ldots \cap E_n = \phi$.

Exhaustive Events

A set of events is said to be **exhaustive**, if atleast one of the events compulsorily occurs.

If a set of events $E_1, E_2, \ldots, E_n$ are exhaustive events, then $E_1 \cup E_2 \cup \ldots \cup E_n = S$

NOTE If the set of events $E_1, E_2, \ldots E_n$ are mutually exclusive and exhaustive events then $E_i \cap E_j = \phi$, $i \neq j$ and $\bigcup_{i=1}^{n} E_i = S$

Complementary Events

In a random experiment, let S be the sample space and E be an event. If $E \subseteq S$, then $E^c = S - E, S$ is called the complement of E.

Probability of an Event

If the sample space has n points (all possible cases) and an event A has m points (all favourable cases), then the probability of A is $P(A) = \frac{m}{n}$.

(i) $0 \le P(A) \le 1$

(ii) $P(S) = 1, P(\phi) = 0$

(iii) Probability of Odds in favour of $A = \frac{P(A)}{P(\bar{A})} = \frac{m}{n-m}$

(iv) Probability of Odds in against $A = \frac{P(\bar{A})}{P(A)} = \frac{n-m}{m}$

Important Results

If n letters corresponding to n envelopes are placed in the envelopes at random, then

(i) probability that letters are in right envelopes $= \frac{1}{n!}$

(ii) probability that letters are not in right envelopes

$$= 1 - \frac{1}{n!}$$

(iii) probability that no letter is in right envelope

$$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots + (-1)^n \frac{1}{n!}$$

(iv) probability that exactly r letters are in right envelopes

$$= \frac{1}{r!}\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots + (-1)^{n-r} \frac{1}{(n-r)!}\right)$$

Addition Theorem of Probability

If A and B are two events associated with a random experiment, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A, B and C are three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$$

For any two events A and B,

$$P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B).$$

Booley's Inequalities

(i) $P(A \cap B) \ge P(A) + P(B) - 1$

(ii) $P(A \cap B \cap C) \ge P(A) + P(B) + P(C) - 2$

NOTE

- $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$
- $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$
- $P(A) = P(A \cap B) + P(A \cap \bar{B})$
- $P(B) = P(B \cap A) + P(B \cap \bar{A})$
- P (exactly one of E_1, E_2 occurs)

  $= P(E_1 \cap E_2') + P(E_1' \cap E_2)$

  $= P(E_1) - P(E_1 \cap E_2) + P(E_2) - P(E_1 \cap E_2)$

  $= P(E_1) + P(E_2) - 2P(E_1 \cap E_2)$
- P (neither E_1 nor E_2) $= P(E_1' \cap E_2') = 1 - P(E_1 \cup E_2)$

Conditional Probability

If A and B are two events associated with the sample space of a random experiment, then conditional probability of the event A, given that B has occurred

i.e. $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}, \; P(B) \ne 0$

Properties of Conditional Probability

Let A and B be two events of a sample space S of an experiment, then

(i) $P\left(\frac{S}{A}\right) = P\left(\frac{A}{A}\right) = 1$ (ii) $P\left(\frac{A'}{B}\right) = 1 - P\left(\frac{A}{B}\right)$,

where A' is complement of A.

Multiplication Theorem of Probability

If A and B are two events associated with a random experiment, then

$$P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right), = P(B) \cdot P\left(\frac{A}{B}\right),$$

$P(A) \ne 0$ and $P(B) \ne 0$

Independent Events

Two events are said to be independent, if the occurrence of one does not depend upon the other.

- If $E_1, E_2, \ldots, E_n$ are independent events, then

  $P(E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_n) = P(E_1) \cdot P(E_2) \ldots P(E_n)$.
- If E and F are independent events, then the pairs E and $\bar{F}$, $\bar{E}$ and F, $\bar{E}$ and $\bar{F}$ are also independent.

Theorem of Total Probability

If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots, B_n$ and the probabilities $P(A/B_1), P(A/B_2), \ldots P(A/B_n)$ are known, then

$$P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)$$

Baye's Theorem

Let the sample space S be the union of n non-empty disjoint subsets (mutually exclusive and exhaustive events).

i.e. $S = A_1 \cup A_2 \cup \ldots \cup A_n$ and $A_i \cap A_j = \phi, i \ne j$

For any event B such that

$$B = (A_1 \cap B) \cup (A_2 \cap B) \cup \ldots \cup (A_n \cap B),$$

$$P(A_i/B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i) P(B/A_i)}{\sum_{i=1}^{n} P(A_i) P(B/A_i)}$$

Probability Distribution of a Random Variable

- A random variable is a real valued function whose domain is the sample space of a random experiment.
- A random variable is usually denoted by the capital letters $X, Y, Z, \dots$ and so on.
- If a random variable X takes values, $x_1, x_2, x_3, \dots, x_n$ with respective probabilities $p_1, p_2, p_3, \dots, p_n$, then

X	x_1	x_2	x_3	...	x_n
$P(X)$	p_1	p_2	p_3	...	p_n

is known as the probability distribution of X.

- The probability distribution of random variable X is defined only when the various values of the random variable, e.g. $x_1, x_2, x_3, \dots, x_n$ together with respective probabilities $p_1, p_2, p_3, \dots, p_n$ satisfy $p_i > 0$ and $\sum_{i=1}^{n} p_i = 1$, where $i = 1, 2, n$

Mean

If a random variable X assumes values $x_1, x_2, x_3, \dots, x_n$ with respective probabilities $p_1, p_2, p_3, \dots, p_n$, then the mean $\overline{X}$ of X is defined as

$$\overline{X} = p_1 x_1 + p_2 x_2 + \dots + p_i x_i \quad \text{or} \quad \overline{X} = \sum_{i=1}^{n} p_i x_i$$

The mean of a random variable X is also known as its mathematical expectation and it is denoted by E (X).

Variance

If a random variable X assumes values $x_1, x_2, x_3, \dots, x_n$ with the respective probabilities $p_1, p_2, \dots, p_n$, then variance of X is given by $\text{Var}(X) = \sum_{i=1}^{n} p_i x_i^2 - \left(\sum_{i=1}^{n} p_i x_i\right)^2$.

Bernoulli Trials and Binomial Distribution

Bernoulli Trials

Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be a finite number of trials
(ii) The trials should be independent
(iii) Each trial has exactly two outcomes; success or failure
(iv) The probability of success (or failure) remains the same in each trial.

Binomial Distribution

The probability of r successes in n independent Bernoulli trials is denoted by $P(X = r)$ and is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

where, p = Probability of success
and q = Probability of failure and $p + q = 1$

(i) Mean $= np$ (ii) Variance $= npq$
(iii) Mean is always greater than variance.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3 is

(a) $\frac{4}{55}$ (b) $\frac{4}{35}$ (c) $\frac{4}{33}$ (d) $\frac{4}{1155}$

2 If the letters of the word 'MATHEMATICS' are arranged arbitrarily, the probability that C comes before E, E before H, H before I and I before S, is

(a) $\frac{1}{75}$ (b) $\frac{1}{24}$ (c) $\frac{1}{120}$ (d) $\frac{1}{720}$

3 If A and B are two events, then the probability that exactly one of them occurs is given by

(a) $P(A) + P(B) - 2P(A \cap B)$
(b) $P(A \cap B') - P(A' \cap B)$
(c) $P(A \cup B) + P(A \cap B)$
(d) $P(A') + P(B') + 2P(A' \cap B')$

4 The probability that atleast one of the events A and B occur is 3/5. If A and B occur simultaneously with probability 1/5, then $P(A') + P(B')$ is equal to

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{6}{5}$ (d) $\frac{7}{5}$

5 A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then, $P(A \cup B)$ is
→ AIEEE 2008

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$
(c) 0 (d) 1

6 For three events, A, B and C, if P (exactly one of A or B occurs) $= P$ (exactly one of B or C occurs) $= P$ (exactly one of C or A occurs) $= \frac{1}{4}$ and P (all the three events occur simultaneously) $= \frac{1}{16}$, then the probability that atleast one of the events occurs, is ➜ JEE Mains 2017

(a) $\frac{7}{32}$ (b) $\frac{7}{16}$ (c) $\frac{7}{64}$ (d) $\frac{3}{16}$

7 In a leap year, the probability of having 53 Sunday or 53 Monday is ➜ NCERT Exemplar

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

8 A number is chosen at random among the first 120 natural numbers. The probability of the number chosen being a multiple of 5 or 15 is

(a) $\frac{1}{8}$ (b) $\frac{1}{5}$ (c) $\frac{1}{24}$ (d) $\frac{1}{6}$

9 Consider two events A and B. If odds against A are as 2 : 1 and those in favour of $A \cup B$ are as 3 : 1, then

(a) $\frac{1}{2} \le P(B) \le \frac{3}{4}$ (b) $\frac{5}{12} \le P(B) \le \frac{3}{4}$

(c) $\frac{1}{4} \le P(B) \le \frac{3}{5}$ (d) None of these

10 A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$. Then, $P(B/A)$ is equal to ➜ NCERT Exemplar

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{10}$ (d) $\frac{1}{5}$

11 If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P\left(\frac{A}{B}\right) = \frac{1}{4}$, then $P(A' \cap B')$ is equal to ➜ NCERT Exemplar

(a) $\frac{1}{12}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{3}{16}$

12 It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$. Then, $P(B)$ is equal to ➜ AIEEE 2008

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

13 Let A and B be two events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$. Then, $P(A'/B')$ equals ➜ NCERT Exemplar

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$

14 If two events A and B are such that $P(A') = 0.3$, $P(B) = 0.4$ and $(A \cap B') = 0.5$, then $P\left(\frac{B}{A \cup B'}\right)$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

15 If C and D are two events such that $C \subset D$ and $P(D) \ne 0$, then the correct statement among the following is

(a) $P(C/D) \ge P(C)$ (b) $P(C/D) < P(C)$

(c) $P(C/D) = \frac{P(D)}{P(C)}$ (d) $P(C/D) = P(C)$

16 Let E and F be two independent events such that $P(E) > P(F)$. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then

(a) $P(E) = \frac{1}{3}, P(F) = \frac{1}{4}$ (b) $P(E) = \frac{1}{2}, P(F) = \frac{1}{6}$

(c) $P(E) = 1, P(F) = \frac{1}{12}$ (d) $P(E) = \frac{1}{3}, P(F) = \frac{1}{2}$

17 An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the sum of the numbers obtained is noted. If the result is a tail, a card from a well-shuffled pack of eleven cards numbered 2, 3, 4, ..., 12 is picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is

(a) $\frac{192}{401}$ (b) $\frac{193}{401}$ (c) $\frac{193}{792}$ (d) $\frac{17}{75}$

18 A bag contains 3 red and 3 white balls. Two balls are drawn one-by-one. The probability that they are of different colours, is

(a) $\frac{3}{10}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) None of these

19 For two events A and B, if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then

(a) A and B are independent (b) $P\left(\frac{A'}{B}\right) = \frac{3}{4}$

(c) $P\left(\frac{B'}{A'}\right) = \frac{1}{2}$ (d) All of these

20 Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then, the events A and B are ➜ JEE Mains 2014

(a) independent but not equally likely
(b) independent and equally likely
(c) mutually exclusive and independent
(d) equally likely but not independent

21 For independent events $A_1, \ldots, A_n \cdot P(A_i) = \frac{1}{i+1}$, $i = 1, 2, .., n$. Then, the probability that none of the events will occur is

(a) $\frac{n}{(n+1)}$ (b) $\frac{(n-1)}{(n+1)}$ (c) $\frac{1}{(n+1)}$ (d) $n + \left(\frac{1}{(n+1)}\right)$

22 Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$. Then,

(a) $P\left(\frac{B}{A}\right) = P(B) - P(A)$ (b) $P(A^c \cup B^c) = P(A^c) + P(B^c)$

(c) $P(A \cup B)^c = P(A^c) \cdot P(B^c)$ (d) $P\left(\frac{A}{B}\right) = P(B/A)$

23 Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is not true?

(a) E_1 and E_2 are independent
(b) E_2 and E_3 are independent
(c) E_1 and E_3 are independent
(d) E_1, E_2 and E_3 are not independent

24 Let A, B, C be three mutually independent events. Consider the two Statements S_1 and S_2

S_1 : A and $B \cup C$ are independent
S_2 : A and $B \cap C$ are independent

Then,

(a) Both S_1 and S_2 are true (b) Only S_1 is true
(c) Only S_2 is true (d) Neither S_1 nor S_2 is true

25 Two independent events namely A and B and the probability that both A and B occurs is $\frac{1}{10}$ and the probability that neither of them occurs is $\frac{3}{10}$. Then, the probability of occurrence of event B is

(a) $\frac{4-\sqrt{7}}{3}$ (b) $\frac{4+\sqrt{7}}{3}$ (c) $\frac{4+\sqrt{6}}{2}$ (d) $\frac{4-\sqrt{6}}{10}$

26 A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is atleast one girl on the committee, the probability that there are exactly 2 girls on the committee, is

➜ NCERT Exemplar

(a) $\frac{7}{99}$ (b) $\frac{13}{99}$ (c) $\frac{14}{99}$ (d) None of these

27 Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only, if the first misses the target. The probability that the target is hit by the second plane, is

(a) 0.06 (b) 0.14 (c) 0.32 (d) 0.7

28 Three machines E_1, E_2 and E_3 in a certain factory produce 50%, 25% and 25%, respectively, of the total daily output of electric tubes. It is known that 4% of the tubes produced on each of machines E_1 and E_2 are defective and that 5% of those produced on E_3 are defective. If on tube is picked up at random from a day's production, the probability that it is defective, is ➜ NCERT Exemplar

(a) 0.025 (b) 0.125 (c) 0.325 (d) 0.0425

29 A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is ➜ JEE Mains 2018

(a) $\frac{3}{10}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{3}{4}$

30 A person goes to office either by car, scooter, bus or train the probabilities of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$, respectively. The probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$, respectively. If he reaches office in time, the probability that he travelled by car is

(a) $\frac{1}{5}$ (b) $\frac{1}{9}$ (c) $\frac{2}{11}$ (d) $\frac{1}{7}$

31 A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, then the probability that the missing card is black, is

(a) $\frac{2}{3}$ (b) $\frac{15}{26}$ (c) $\frac{16}{39}$ (d) $\frac{37}{52}$

32 A discrete random variable X has the following probability distribution.

X	1	2	3	4	5	6	7
$P(X)$	C	$2C$	$2C$	$3C$	C^2	$2C^2$	$7C^2+C$

The value of C and the mean of the distribution are

➜ NCERT Exemplar

(a) $\frac{1}{10}$ and 3.66 (b) $\frac{1}{20}$ and 2.66
(c) $\frac{1}{15}$ and 1.33 (d) None of these

33 For a random variable $X, E(X)=3$ and $E(X^2)=11$. Then, variable of X is

(a) 8 (b) 5 (c) 2 (d) 1

34 A random variable X has the probability distribution

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, then the probability $P(E \cup F)$ is

(a) 0.87 (b) 0.77 (c) 0.35 (d) 0.50

35 Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards, then the mean of the number of aces is

(a) $\frac{1}{13}$ (b) $\frac{3}{13}$ (c) $\frac{2}{13}$ (d) None of these

36 Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of atleast one failure is greater than or equal to 31/32, then p lies in the interval

(a) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (b) $\left[0, \frac{1}{2}\right]$ (c) $\left(\frac{11}{12}, 1\right]$ (d) $\left(\frac{1}{2}, \frac{3}{4}\right]$

37 A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to getting 9 heads, then the probability of getting 2 heads is

(a) $\frac{15}{2^8}$ (b) $\frac{2}{15}$ (c) $\frac{15}{2^{13}}$ (d) None of these

38 A fair coin is tossed n times. If X is the number of times heads occur and $P(X=4), P(X=5)$ and $P(X=6)$ are in AP, then n is equal to

(a) 13 (b) 7 (c) 11 (d) None of these

39 A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn one-by-one with replacement, then the variance of the number of green balls drawn is

→ JEE Mains 2017

(a) $\frac{12}{5}$ (b) 6 (c) 4 (d) $\frac{6}{25}$

40 A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is → JEE Mains 2013

(a) $\frac{17}{3^5}$ (b) $\frac{13}{3^5}$ (c) $\frac{11}{3^5}$ (d) $\frac{10}{3^5}$

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 7 white balls and 3 black balls are placed in a row at random. The probability that no two black balls are adjacent is

(a) $\frac{1}{2}$ (b) $\frac{7}{15}$ (c) $\frac{2}{15}$ (d) $\frac{1}{3}$

2 Events A, B and C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}, P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$. Then, the set of possible values of x are in the interval

(a) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (b) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (c) $\left[\frac{1}{3}, \frac{13}{3}\right]$ (d) $[0, 1]$

3 If 12 balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls, is → JEE Mains 2015

(a) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (b) $55\left(\frac{2}{3}\right)^{10}$ (c) $220\left(\frac{1}{3}\right)^{12}$ (d) $22\left(\frac{1}{3}\right)^{11}$

4 If 6 objects are distributed at random among 6 persons, the probability that atleast one person does not get any object is

(a) $\frac{313}{324}$ (b) $\frac{315}{322}$ (c) $\frac{317}{324}$ (d) $\frac{319}{324}$

5 10 apples are distributed at random among 6 persons. The probability that atleast one of them will receive none is

(a) $\frac{6}{143}$ (b) $\frac{^{14}C_4}{^{15}C_5}$ (c) $\frac{137}{143}$ (d) $\frac{135}{143}$

6 A draws two cards at random from a pack of 52 cards. After returning them to the pack and shuffling it, B draws two cards at random. The probability that their draws contain exactly one common card is

(a) $\frac{25}{546}$ (b) $\frac{50}{663}$ (c) $\frac{25}{663}$ (d) None of these

7 In a dice game, a player pays a stake of ₹ 1 for each throw of a die. She receives ₹ 5, if the die shows a 3, ₹ 2, if the die shows a 1 or 6 and nothing otherwise. What is the player's expected profit per throw over a long series of throws? → NCERT Exemplar

(a) 0.50 (b) 0.20 (c) 0.70 (d) 0.90

8 In an examination, 20 questions of true-false type are asked. Suppose a students tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers true, if it falls tails, he answers false. The probability that he answers atleast 12 questions correctly is

(a) $\left(\frac{1}{2}\right)^{20}({}^{20}C_{12}+{}^{20}C_{13}+\ldots+{}^{20}C_{20})$

(b) $\left(\frac{1}{2}\right)^{10}({}^{20}C_{11}+{}^{20}C_{12}+\ldots+{}^{20}C_{20})$

(c) $\left(\frac{1}{2}\right)^{20}({}^{20}C_{11}+{}^{20}C_{12}+\ldots+{}^{20}C_{20})$

(d) None of the above

9 In a multiple choice question there are four alternative answers of which one or more than one is correct. A candidate will get marks on the question only if he ticks the correct answer. The candidate decides to tick answers at random. If he is allowed up to three chances of answer the question, then the probability that he will get marks on it is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{5}$ (d) $\frac{2}{15}$

10 Three natural numbers are taken at random from the set $A=\{x \mid 1 \le x \le 100, x \in N\}$. The probability that the AM of the numbers taken is 25, is

(a) $\frac{^{77}C_2}{^{100}C_3}$ (b) $\frac{^{25}C_2}{^{100}C_3}$ (c) $\frac{^{74}C_{72}}{^{100}C_{97}}$ (d) $\frac{^{75}C_2}{^{100}C_3}$

11 Given that $x \in [0,1]$ and $y \in [0,1]$. Let A be the event of (x, y) satisfying $y^2 \le x$ and B be the event of (x, y) satisfying $x^2 \le y$. Then,

(a) $P(A \cap B)=\frac{1}{3}$ (b) A, B are exhaustive

(c) A, B are mutually exclusive (d) A, B are independent

12 If two different numbers are taken from the set $\{0, 1, 2, 3, \ldots, 10\}$, then the probability that their sum as well as absolute difference are both multiple of 4, is

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(a) $\frac{6}{55}$ (b) $\frac{12}{55}$ (c) $\frac{14}{45}$ (d) $\frac{7}{55}$

13 One Indian and four American men and their wives are to be seated randomly around a circular table. Then, the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

14 If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7^n + 7^m$ is divisible by 5 equals

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) $\frac{1}{3}$

ANSWERS

SESSION 1	**1** (d)	**2** (c)	**3** (a)	**4** (c)	**5** (d)	**6** (b)	**7** (b)	**8** (b)	**9** (b)	**10** (b)
	11 (c)	**12** (c)	**13** (c)	**14** (a)	**15** (a)	**16** (a)	**17** (c)	**18** (c)	**19** (d)	**20** (a)
	21 (c)	**22** (c)	**23** (d)	**24** (a)	**25** (d)	**26** (d)	**27** (c)	**28** (d)	**29** (b)	**30** (d)
	31 (a)	**32** (a)	**33** (c)	**34** (b)	**35** (c)	**36** (b)	**37** (c)	**38** (b)	**39** (a)	**40** (c)
SESSION 2	**1** (b)	**2** (a)	**3** (a)	**4** (d)	**5** (c)	**6** (b)	**7** (a)	**8** (a)	**9** (c)	**10** (c)
	11 (a)	**12** (a)	**13** (c)	**14** (a)						

Hints and Explanations

SESSION 1

1 Here, $n(S) = {}^{100}C_3$

Let E = All three of them are divisible by both 2 and 3.

$\Rightarrow$ Divisible by 6 i.e. $\{6, 12, 18, \ldots, 96\}$

Thus, out of 16 we have to select 3.

$\therefore \quad n(E) = {}^{16}C_3$

$\therefore$ Required probability $= \frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$

2 Total number of arrangement is

$$\frac{11!}{2!2!2!} = \frac{11!}{8}$$

Number of arrangement in which C, E, H, I and S appear in that order

$$= ({}^{11}C_5)\frac{6!}{2!2!2!} = \frac{11!}{8.5!}$$

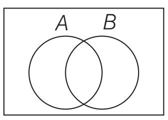
$\therefore$ Required probability

$$= \frac{11!}{8.5!} \div \frac{11!}{8} = \frac{1}{5!} = \frac{1}{120}$$

3 P(Exactly one of the events occurs)

$= P(A \cap B') \cup (B \cap A')$

$= P(A \cap B') + P(B \cap A')$



$= P(A) + P(B) - 2P(A \cap B)$

4 Here, $P(A \cup B) = \frac{3}{5}$ and $P(A \cap B) = \frac{1}{5}$

From addition theorem, we get

$$\frac{3}{5} = P(A) + P(B) - \frac{1}{5}$$

$$\Rightarrow \quad \frac{4}{5} = 1 - P(A') + 1 - P(B')$$

$$\therefore \quad P(A') + P(B') = 2 - \frac{4}{5} = \frac{6}{5}$$

5 Clearly, $A = \{4, 5, 6\}$ and $B = \{1, 2, 3, 4\}$

$\therefore A \cap B = \{4\}$

Now, by addition theorem of probability

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1$

6 We have, P (exactly one of A or B occurs) $= P(A \cup B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

According to the question,

$$P(A) + P(B) - 2P(A \cap B) = \frac{1}{4} \quad \ldots\text{(i)}$$

$$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4} \quad \ldots\text{(ii)}$$

and $P(C) + P(A) - 2P(C \cap A) = \frac{1}{4} \quad \ldots$(iii)

On adding Eqs. (i), (ii) and (iii), we get

$$2\,[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = \frac{3}{4}$$

$$\Rightarrow \quad P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8}$$

$\therefore$ P (atleast one event occurs)

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}\left[\because P(A \cap B \cap C) = \frac{1}{16}\right]$$

7 Since, a leap year has 366 days and hence 52 weeks and 2 days. The 2 days can be (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun). Therefore,

$$P\text{(53 Sunday or 53 Monday)} = \frac{3}{7}$$

8 In first 120 natural numbers, total number of multiples of 5, $n(A) = 24$ and total number of multiples of 15, $n(B) = 8$ and $n(A \cap B) = 8$.

$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 24 + 8 - 8 = 24$

$\therefore$ Required probability $= \frac{24}{120} = \frac{1}{5}$

9 Here, $P(A) = \frac{1}{3}$ and $P(A \cup B) = \frac{3}{4}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\leq P(A) + P(B)$

$\Rightarrow \frac{3}{4} \leq \frac{1}{3} + P(B) \Rightarrow P(B) \geq \frac{5}{12}$

Also, $B \subseteq A \cup B$

$\Rightarrow P(B) \leq P(A \cup B) = \frac{3}{4}$

$\therefore \frac{5}{12} \leq P(B) \leq \frac{3}{4}$

10 Given, $P(A) = 0.4$, $P(B) = 0.3$, $P(A \cup B) = 0.5$

$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{P(A) + P(B) - P(A \cup B)}{P(A)} = \frac{0.4 + 0.3 - 0.5}{0.4} = \frac{1}{2}$

11 Given, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$

and $P\left(\frac{A}{B}\right) = \frac{1}{4}$

$\therefore P(A' \cap B') = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - P(A) - P(B) + P\left(\frac{A}{B}\right) \cdot P(B)$

$= 1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{3} = 1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{12}$

$= \frac{12 - 6 - 4 + 1}{12} = \frac{3}{12} = \frac{1}{4}$

12 Given that, $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$

and $P\left(\frac{B}{A}\right) = \frac{2}{3}$

We know that,

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

and $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$\therefore P(B) = \frac{P\left(\frac{B}{A}\right) \cdot P(A)}{P\left(\frac{A}{B}\right)} = \frac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$

13 Given that, $P(A) = 0.6$, $P(B) = 0.2$

$P(A/B) = 0.5$

$P(A \cap B) = P(A/B) \cdot P(B) = (0.5)(0.2) = 0.1$

$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{P[(A \cup B)']}{P(B')}$

$= \frac{1 - P(A \cup B)}{1 - P(B)}$

$= \frac{1 - P(A) - P(B) + P(A \cap B)}{1 - 0.2} = \frac{3}{8}$

14 $P(B/A \cup B') = \frac{P\{B \cap (A \cup B')\}}{P(A \cup B')}$

$= \frac{P(A \cap B)}{P(A) + P(B') - P(A \cap B')}$

$= \frac{P(A) - P(A \cap B')}{0.7 + 0.6 - 0.5} = \frac{0.7 - 0.5}{0.8} = \frac{1}{4}$

15 As, $P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)}$ $\left[\begin{array}{l}\because C \subset D \\ \therefore P(C \cap D) = P(C)\end{array}\right]$

$= \frac{P(C)}{P(D)}$ ...(i)

Also, as $P(D) \leq 1$

$\therefore \frac{1}{P(D)} \geq 1$ and $\frac{P(C)}{P(D)} \geq P(C)$ ...(ii)

From Eqs. (i) and (ii), we get

$P\left(\frac{C}{D}\right) = \frac{P(C)}{P(D)} \geq P(C)$

16 $P(E \cap F) = P(E)\,P(F) = \frac{1}{12}$ ...(i)

$P(E^C \cap F^C) = P(E^C) \cdot P(F^C) = \frac{1}{2}$

$\Rightarrow (1 - P(E))(1 - P(F)) = \frac{1}{2}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$P(E) = \frac{1}{3}$ and $P(F) = \frac{1}{4}$,

as $P(E) > P(F)$

17 The probability of getting the sum 7 or 8 from two dice is $\frac{6}{36} + \frac{5}{36} = \frac{11}{36}$.

The probability of getting the card with number 7 or 8 is $\frac{2}{11}$.

$\therefore$ Required probability

$= \frac{1}{2} \cdot \frac{11}{36} + \frac{1}{2} \cdot \frac{2}{11} = \frac{11}{72} + \frac{2}{22} = \frac{193}{792}$

18 Let A be event that drawn ball is red and B be event that drawn ball is white. Then, AB and BA are two disjoint cases of the given event.

$\therefore P(AB + BA) = P(AB) + P(BA)$

$= P(A)\,P\left(\frac{B}{A}\right) + P(B) \cdot P\left(\frac{A}{B}\right)$

$= \frac{3}{6} \cdot \frac{3}{5} + \frac{3}{6} \cdot \frac{3}{5} = \frac{3}{5}$

19 Given that,

$P\left(\frac{B}{A}\right) = \frac{1}{2} \Rightarrow \frac{P(B \cap A)}{P(A)} = \frac{1}{2}$

$\Rightarrow P(B \cap A) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{4}$

$\Rightarrow P(B) = 4P(A \cap B) \Rightarrow P(B) = \frac{1}{2}$

$\because P(A \cap B) = \frac{1}{8} = \frac{1}{2} \cdot \frac{1}{4} = P(A) \cdot P(B)$

$\therefore$ Events A and B are independent.

Now, $P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)}$

$= \frac{P(A')\,P(B)}{P(B)} = \frac{3}{4}$

and $P\left(\frac{B'}{A'}\right) = \frac{P(B' \cap A')}{P(A')}$

$= \frac{P(B')\,P(A')}{P(A')} = \frac{1}{2}$

20 Given, $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$

and $P(\bar{A}) = \frac{1}{4}$

$\therefore P(A \cup B) = 1 - P(\overline{A \cup B})$

$= 1 - \frac{1}{6} = \frac{5}{6}$

and $P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4} \Rightarrow P(B) = \frac{1}{3}$

$\Rightarrow A$ and B are not equally likely.

Also, $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4}$

So, events are independent.

21 P (non-occurrence of (A_i))

$= 1 - \frac{1}{(i+1)} = \frac{i}{(i+1)}$

$\therefore P$ (non-occurrence of any of events)

$= \left(\frac{1}{2}\right) \cdot \left(\frac{2}{3}\right) \cdots \left\{\frac{n}{(n+1)}\right\} = \frac{1}{(n+1)}$

22 $P(A \cap B) = P(A) \cdot P(B)$

It means A and B are independent events, so A^C and B^C will also be independent.

Hence, $P(A \cup B)^C = P(A^C \cap B^C)$ [De-Morgan's law]

$= P(A^C)\,P(B^C)$

As A is independent of B,

$P\left(\frac{A}{B}\right) = P(A)$

$\left[\because P(A \cap B) = P(B) \cdot P\left(\frac{A}{B}\right)\right]$

23 Clearly, $E_1 = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}$

$E_2 = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)\}$

and $E_3 = \{(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4, 5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)\}$

$\Rightarrow P(E_1) = \frac{6}{36} = \frac{1}{6}$, $P(E_2) = \frac{6}{36} = \frac{1}{6}$

and $P(E_3) = \frac{18}{36} = \frac{1}{2}$

Now, $P(E_1 \cap E_2) = P$

[getting 4 on die A and 2 on die B]

$= \frac{1}{36} = P(E_1) \cdot P(E_2)$

$P(E_2 \cap E_3) = P$

[getting 2 on die B and sum of numbers on both dice is odd]

$= \frac{3}{36} = P(E_2) \cdot P(E_3)$

$P(E_1 \cap E_3) = P$ [getting 4 on die A and sum of numbers on both dice is odd]

$$= \frac{3}{36} = P(E_1)\cdot P(E_3)$$

and $P(E_1 \cap E_2 \cap E_3) = P$ [getting 4 on die A, 2 on die B and sum of numbers is odd]

$= P$ (impossible event) $= 0$

Hence, E_1, E_2 and E_3 are not independent.

24 Consider,

$$P\{A \cap (B \cap C)\} = P(A \cap B \cap C)$$
$$= P(A)\cdot P(B)\cdot P(C) = P(A)\cdot (B \cap C)$$

Now, consider

$$P[A \cap (B \cup C)] = P[(A \cap B) \cup (A \cap C)]$$
$$= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$$
$$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$
$$= P(A)\cdot P(B) + P(A)\,P(C) - P(A)\,P(B)\,P(C)$$
$$= P(A)[P(B) + P(C) - P(B)\,P(C)]$$
$$= P(A)\cdot P(B \cup C)$$

$B \cup C$ is independent to A, so S_1 is true.

$B \cap C$ is also independent to A, so S_2 is true.

25 We have,

$P(A \cap B) = \frac{1}{10}$ and $P(\overline{A \cup B}) = \frac{3}{10}$.

Then, $P(A \cup B) = 1 - P(\overline{A \cup B}) = \frac{7}{10}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad P(A) + P(B) = P(A \cup B) + P(A \cap B)$

$$\Rightarrow \quad P(A) + P(B) = \frac{7}{10} + \frac{1}{10} = \frac{4}{5} \quad \ldots(i)$$

$$\because \quad P(A)\cdot P(B) = \frac{1}{10}$$

$$\Rightarrow \quad P(A) = \frac{1}{10P(B)}$$

From Eq. (i), $P(B) + \frac{1}{10\{P(B)\}} = \frac{4}{5}$

$$\Rightarrow \quad 10\{P(B)\}^2 + 1 = 8P(B)$$

Let $P(B) = t$, then $10t^2 + 1 = 8t$

$$\Rightarrow \quad 10t^2 - 8t + 1 = 0$$

$$\therefore \quad t = \frac{8 \pm \sqrt{64 - 4 \times 10 \times 1}}{2 \times 10} = \frac{8 \pm 2\sqrt{6}}{20}$$

So, $P(B) = \frac{4 - \sqrt{6}}{10}$ is possible.

26 Let A denote the event that atleast one girl will be chosen and B the event that exactly 2 girls will be chosen. We require $P(B|A)$.

Since, A denotes the event that atleast one girl will be chosen. A denotes that no girl is chosen i.e. 4 boys are chosen. Then,

$$P(A') = \frac{{}^8C_4}{{}^{12}C_4} = \frac{70}{495} = \frac{14}{99}$$

$$P(A) = 1 - \frac{14}{99} = \frac{85}{99}$$

Now, $P(A \cap B) = P$ (2 boys and 2 girls)

$$= \frac{{}^8C_2 \cdot {}^4C_2}{{}^{12}C_4} = \frac{6 \times 28}{495} = \frac{56}{165}$$

Thus, $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$$= \frac{56}{165} \times \frac{99}{85} = \frac{168}{425}$$

27 Let the events be A = Ist aeroplane hit the target

B = IInd aeroplane hit the target

and their corresponding probabilities are

$P(A) = 0.3$ and $P(B) = 0.2$

$\Rightarrow \quad P(\bar{A}) = 0.7$ and $P(\bar{B}) = 0.8$

$\therefore$ Required probability

$$= P(\bar{A})P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(B) + \ldots$$
$$= (0.7)(0.2) + (0.7)(0.8)(0.7)(0.2) + (0.7)(0.8)(0.7)(0.8)(0.7)(0.2) + \ldots$$
$$= 0.14\,[1 + (0.56) + (0.56)^2 + \ldots]$$
$$= 0.14\left(\frac{1}{1 - 0.56}\right) = \frac{0.14}{0.44} = 0.32$$

28 Let D be the event that the picked up tube is defective.

Let A_1, A_2 and A_3 be the events that the tube is produced on machines E_1, E_2 and E_2, respectively.

$$P(D) = P(A_1)\,P(D \mid A_1) + P(A_2)\,P(D \mid A_2) + P(A_3)\,P(D \mid A_3) \ldots(i)$$

$$P(A_1) = \frac{50}{100} = \frac{1}{2},\ P(A_2) = \frac{1}{4},\ P(A_3) = \frac{1}{4}$$

Also, $P(D \mid A_1) = P(D \mid A_2) = \frac{4}{100} = \frac{1}{25}$

$$P(D \mid A_3) = \frac{5}{100} = \frac{1}{20}.$$

On putting these values in Eq. (i), we get

$$P(D) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$
$$= \frac{1}{50} + \frac{1}{100} + \frac{1}{80} = \frac{17}{400} = 0.0425$$

29 **Key idea** Use the theorem of total probability

Let E_1 = Event that first ball drawn is red

E_2 = Event that first ball drawn is black

A = Event that second ball drawn is red

$$P(E_1) = \frac{4}{10},\ P\left(\frac{A}{E_1}\right) = \frac{6}{12}$$

$$\Rightarrow \quad P(E_2) = \frac{6}{10},\ P\left(\frac{A}{E_2}\right) = \frac{4}{12}$$

By law of total probability

$$P(A) = P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)$$

$$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12} = \frac{24 + 24}{120}$$
$$= \frac{48}{120} = \frac{2}{5}$$

30 Let C, S, B and T be the events of the person going by car, scooter, bus and train, respectively.

$$P(C) = \frac{1}{7},\ P(S) = \frac{3}{7},\ P(B) = \frac{2}{7},\ P(T) = \frac{1}{7}$$

Let L be the event that the person reaching the office in time. Then, $\bar{L}$ be the event that the person reaching the office in late.

$$P(L/C) = \frac{7}{9},\ P(L/S) = \frac{8}{9},$$
$$P(L/B) = \frac{5}{9},\ P(L/T) = \frac{8}{9}$$

$\therefore$ Required probability =

$$P(C/L) = \frac{P(C)\cdot P(L/C)}{\left[\begin{array}{l} P(C)\cdot P(L/C) \\ + P(S)\cdot P(L/S) \\ + P(B)\cdot P(L/B) \\ + P(T)\cdot P(L/T) \end{array}\right]}$$

$$= \frac{\frac{1}{7}\cdot\frac{7}{9}}{\frac{1}{7}\cdot\frac{7}{9} + \frac{3}{7}\cdot\frac{8}{9} + \frac{2}{7}\cdot\frac{5}{9} + \frac{1}{7}\cdot\frac{8}{9}} = \frac{7}{49} = \frac{1}{7}$$

31 Let B stand for the event that black card is missing, then $P(B) = P(\bar{B}) = \frac{1}{2}$.

Let E be the event that all the first 13 cards are red.

$$\therefore\ P(E/B) = \frac{26}{51}\cdot\frac{25}{50}\ldots\frac{14}{39},$$

$$P(E/\bar{B}) = \frac{25}{51}\cdot\frac{24}{50}\ldots\frac{13}{39}$$

$$P(B/E) = \frac{P(B)\cdot P(E/B)}{P(B)\cdot P(E/B) + P(\bar{B})\cdot P(E/\bar{B})}$$

$$= \frac{26\cdot 25 \ldots 14}{26\cdot 25 \ldots 14 + 25\cdot 24 \ldots 13}$$

$$= \frac{26}{26 + 13} = \frac{26}{39} = \frac{2}{3}$$

32 Since, $\Sigma P_i = 1$, we have

$$C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$$

i.e. $\quad 10C^2 + 9C - 1 = 0$

i.e. $\quad (10C - 1)(C + 1) = 0$

$$\Rightarrow \quad C = \frac{1}{10},\ C = -1$$

Therefore, the permissible value of $C = \frac{1}{10}$

$$\text{Mean} = \sum_{i=1}^{n} x_i p_i = \sum_{i=1}^{7} x_i p_i$$

$$= 1 \times \frac{1}{10} + 2 \times \frac{2}{10} + 3 \times \frac{2}{10} + 4 \times \frac{3}{10} + 5\left(\frac{1}{10}\right)^2$$

$$+ 6 \times 2\left(\frac{1}{10}\right)^2 + 7\left(7\left(\frac{1}{10}\right)^2 + \frac{1}{10}\right)$$

$$= \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{12}{10} + \frac{5}{100} + \frac{12}{100} + \frac{49}{100} + \frac{7}{10}$$

$= 3.66$

33 Given that, $E(X) = 3$ and $(E(X^2)) = 11$

Variance of $X = E(X^2) - [E(X)]^2$

$= 11 - (3)^2 = 11 - 9 = 2$

34 Given, $E = \{X \text{ is a prime number}\}$

$= \{2, 3, 5, 7\}$

$\therefore \quad P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)$

$\Rightarrow \quad P(E) = 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

and $\quad F = \{X < 4\} = \{1, 2, 3\}$

$\Rightarrow P(F) = P(X = 1) + P(X = 2) + P(X = 3)$

$\Rightarrow P(F) = 0.15 + 0.23 + 0.12 = 0.5$

and $E \cap F = \{X$ is prime number as well as $< 4\} = \{2, 3\}$

$P(E \cap F) = P(X = 2) + P(X = 3)$

$= 0.23 + 0.12 = 0.35$

$\therefore$ Required probability

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.62 + 0.5 - 0.35 = 0.77$

35 Let X denote the number of aces.

Probability of selecting a ace,

$$p = \frac{4}{52} = \frac{1}{13}$$

And probability of not selecting ace,

$$q = 1 - \frac{1}{13} = \frac{12}{13}$$

$$P(X = 0) = \left(\frac{12}{13}\right)^2,$$

$$P(X = 1) = 2 \cdot \left(\frac{1}{13}\right) \cdot \left(\frac{12}{13}\right) = \frac{24}{169}$$

$$P(X = 2) = \left(\frac{1}{13}\right)^2 \cdot \left(\frac{12}{13}\right)^0 = \frac{1}{169}$$

$$\text{Mean} = \Sigma P_i X_i = 0 + 1 \times \frac{24}{169} + 2 \times \frac{1}{169} = \frac{24}{169} + \frac{2}{169} = \frac{2}{13}$$

36 We have, $n = 5$ and $r \geq 1$

$\therefore \quad P(X = r) = {}^nC_r p^{n-r} q^r$,

$\therefore \quad P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^5C_0 . p^5 . q^0 \geq \frac{31}{32}$$

$$\Rightarrow \quad p^5 \leq 1 - \frac{31}{32} = \frac{1}{32}$$

$$\therefore \quad p \leq \frac{1}{2} \text{ and } p \geq 0 \Rightarrow p \in \left[0, \frac{1}{2}\right]$$

37 Let the coin was tossed n times and X be the random variable representing the number of head appearing in n trials.

According to the given condition,

$\therefore \quad P(X = 7) = P(X = 9)$

$$\Rightarrow {}^nC_7 \cdot \left(\frac{1}{2}\right)^7 \cdot \left(\frac{1}{2}\right)^{n-7} = {}^nC_9 \cdot \left(\frac{1}{2}\right)^9 \cdot \left(\frac{1}{2}\right)^{n-9}$$

$\Rightarrow \quad {}^nC_7 = {}^nC_9 \Rightarrow n = 16$

$[\because {}^nC_x = {}^nC_y \Rightarrow x + y = n]$

$$\therefore \quad P(X = 2) = {}^{16}C_2 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{14}$$

$$= \frac{{}^{16}C_2}{2^{16}} = \frac{16 \cdot 15}{2^{17}} = \frac{15}{2^{13}}$$

38 Since, ${}^nC_4 \frac{1}{2^n}, {}^nC_5 \frac{1}{2^n}$ and ${}^nC_6 \frac{1}{2^n}$ are in AP.

$\therefore \quad {}^nC_4, {}^nC_5$ and ${}^nC_6$ are also in AP.

$\therefore \quad 2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$

$\therefore$ On dividing by ${}^nC_5$ both sides, we get

$$2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5} = \frac{5}{n-4} + \frac{n-5}{6}$$

$$= \frac{n^2 - 9n + 50}{6(n-4)}$$

$\Rightarrow \quad n^2 - 21n + 98 = 0$

$\Rightarrow \quad n = 7, 14$

39 Given box contains 15 green and 10 yellow balls.

$\therefore$ Total number of balls $= 15 + 10 = 25$

$$P(\text{green balls}) = \frac{15}{25} = \frac{3}{5} = p$$

$=$ Probability of success

$$P(\text{yellow balls}) = \frac{10}{25} = \frac{2}{5} = q$$

$=$ Probability of failure

and $n = 10 =$ Number of trials.

$$\therefore \text{ Variance} = npq = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$$

40 Clearly, probability of guessing correct answer, $p = \frac{1}{3}$ and probability of guessing a wrong answer, $q = \frac{2}{3}$

$\therefore$ The probability of guessing a 4 or more correct answer

$$= {}^5C_4 \left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + {}^5C_5 \left(\frac{1}{3}\right)^5$$

$$= 5 \cdot \frac{2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$$

SESSION 2

1 $\overset{B}{-}W\overset{B}{-}W\overset{B}{-}W\overset{B}{-}W\overset{B}{-}W\overset{B}{-}W\overset{B}{-}W\overset{B}{-}$

$$n(S) = \frac{10!}{(7!)(3!)} \quad n(E) = {}^8C_3 = \frac{8!}{(3!)(5!)}$$

[because there are 8 places for 3 black balls]

$$\therefore \quad P(E) = \frac{\frac{8!}{(3!)(5!)}}{\frac{10!}{(7!)(3!)}} = \frac{(8!)(7!)}{(10!)(5!)}$$

$$= \frac{7 \cdot 6}{10 \cdot 9} = \frac{7}{15}$$

2 Since, $0 \leq P(A) \leq 1$, $0 \leq P(B) \leq 1$, $0 \leq P(C) \leq 1$

and $0 \leq P(A) + P(B) + P(C) \leq 1$

$$\therefore \quad 0 \leq \frac{3x+1}{3} \leq 1 \Rightarrow -\frac{1}{3} \leq x \leq \frac{2}{3} \quad \ldots\text{(i)}$$

$$0 \leq \frac{1-x}{4} \leq 1 \Rightarrow -3 \leq x \leq 1 \quad \ldots\text{(ii)}$$

$$0 \leq \frac{1-2x}{2} \leq 1 \Rightarrow -\frac{1}{2} \leq x \leq \frac{1}{2} \quad \ldots\text{(iii)}$$

$$\text{and } 0 \leq \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \leq 1$$

$\Rightarrow \quad 0 \leq 13 - 3x \leq 12$

$$\Rightarrow \quad \frac{1}{3} \leq x \leq \frac{13}{3} \quad \ldots\text{(iv)}$$

From Eqs. (i), (ii), (iii) and (iv), we get

$$\frac{1}{3} \leq x \leq \frac{1}{2}$$

3 There seems to be ambiguity in this question. It should be mentioned that boxes are different and one particular box has 3 balls. Then, the required probability

$$= \frac{{}^{12}C_3 \times 2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$$

4 Number of ways of distributing 6 objects to 6 persons $= 6^6$

Number of ways of distributing 1 object to each person $= 6!$

$\therefore$ Required probability

$$= 1 - \frac{6!}{6^6} = 1 - \frac{5!}{6^5} = \frac{319}{324}$$

5 The required probability

$= 1 -$ probability of each receiving atleast one $= 1 - \frac{n(E)}{n(S)}$

Now, the number of integral solutions of

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$$

such that $x_1 \geq 1, x_2 \geq 1, \ldots, x_6 \geq 1$ gives $n(E)$ and the number of integral solutions of $x_1 + x_2 + \ldots + x_5 + x_6 = 10$ such that $x_1 \geq 0, x_2 \geq 0, \ldots, x_6 \geq 0$ gives $n(S)$.

$\therefore$ The required probability

$$= 1 - \frac{{}^{10-1}C_{6-1}}{{}^{10+6-1}C_{6-1}} = 1 - \frac{{}^9C_5}{{}^{15}C_5} = \frac{137}{143}$$

6 The probability of both drawing the common card x, $P(X) =$ (Probability of A drawing the card x and any other card y) $\times$ (Probability of B drawing the card x and a card other than y)

$$= \frac{{}^{51}C_1}{{}^{52}C_2} \times \frac{{}^{50}C_1}{{}^{52}C_2} \ \forall\ x,$$

where x has 52 values.

$\therefore$ Required probability $= \Sigma P(X)$

$$= 52 \times \frac{51 \times 50 \times 4}{52 \times 51 \times 52 \times 51} = \frac{50}{663}$$

7 Let X be the money won in one throw.
Money lost in 1 throw = ₹ 1

Also, probability of getting 3 = $\frac{1}{6}$

Probability of getting 1 or 6

$$\Rightarrow \quad \frac{1}{6}+\frac{1}{6}=\frac{2}{6}$$

Probability of getting any other number i.e. 2 or 4 or 5

$$=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}$$

Then, probability distribution is

X	5	2	0
$P(X)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

Then, expected money that player can be won

$$E(X)=\frac{5}{6}+\frac{4}{6}+0=\frac{9}{6}=₹\ 1.5$$

Thus, player's expected profit

$$= ₹\ 1.5 - ₹\ 1 = 0.50$$

8 Let X denote the number of correct answer given by the student. The repeated tosses of a coin are Bernoulli trials. Since, head on a coin represent the true answer and tail represents the false answer, the correctly answered of the question are Bernoulli trials.

$\therefore\ p = P(\text{a success}) = P(\text{coin show up a head}) = \frac{1}{2}$ and $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

So, X has a binomial distribution with $n = 20,\ p = \frac{1}{2}$ and $q = \frac{1}{2}$

$$\therefore\ P(X=r)={}^{20}C_r\cdot\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{20-r}$$

Hence, P (atleast 12 questions are answered as true)

$$= P(X \ge 12) = P(12) + P(13) + P(14) + P(15) + P(16) + P(17) + P(18) + P(19) + P(20)$$

$$= {}^{20}C_{12}\ p^{12}q^{8} + {}^{20}C_{13}\ p^{13}q^{7} + {}^{20}C_{14}\ p^{14}q^{6} + {}^{20}C_{15}\ p^{15}q^{5} + {}^{20}C_{16}\ p^{16}q^{4} + {}^{20}C_{17}\ p^{17}q^{3} + {}^{20}C_{18}\ p^{18}q^{2} + {}^{20}C_{19}\ p^{19}q^{1} + {}^{20}C_{20}\ p^{20}$$

$$= ({}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20})\cdot\frac{1}{2^{20}}$$

$$= \left(\frac{1}{2}\right)^{20}({}^{20}C_{12} + {}^{20}C_{13} + \ldots + {}^{20}C_{20})$$

9 The total number of ways of ticking one or more alternatives out of 4 is ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 15$. Out of these 15 combinations only one combination is correct. The probability of ticking the alternative correctly at the first trial is $\frac{1}{15}$ that at the second trial is $\left(\frac{14}{15}\right)\left(\frac{1}{14}\right)=\frac{1}{15}$ and that at the third trial is

$$\left(\frac{14}{15}\right)\left(\frac{13}{14}\right)\left(\frac{1}{13}\right)=\frac{1}{15}$$

Thus, the probability that the candidate will get marks on the question, if he is allowed up to three trials is

$$\frac{1}{15}+\frac{1}{15}+\frac{1}{15}=\frac{1}{5}$$

10 Here, $n(S) = {}^{100}C_3$

As the AM of three numbers is 25, their sum = 75

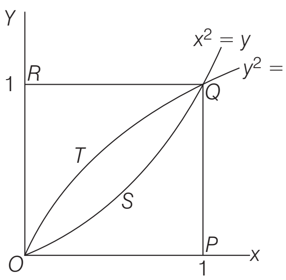
$\therefore\ n(E)$ = the number of integral solutions of $x_1 + x_2 + x_3 = 75$, where, $x_1 \ge 1,\ x_2 \ge 1,\ x_3 \ge 1$

$${}^{75-1}C_{3-1} = {}^{74}C_2 = {}^{74}C_{72}$$

$$\therefore\ P(E)=\frac{{}^{74}C_{72}}{{}^{100}C_3}=\frac{{}^{74}C_{72}}{{}^{100}C_{97}}$$

11 A = the event of (x, y) belonging to the area $OTQPO$

B = the event of (x, y) belonging to the area $OSQRO$



$$P(A)=\frac{\text{ar}(OTQPO)}{\text{ar}(OPQRO)}=\frac{\int_0^1\sqrt{x}\,dx}{1\times1}=\left[\frac{2}{3}x^{3/2}\right]_0^1=\frac{2}{3}$$

$$P(B)=\frac{\text{ar}(OSQRO)}{\text{ar}(OPQRO)}=\frac{\int_0^1\sqrt{y}\,dy}{1\times1}=\frac{2}{3}$$

$$P(A\cap B)=\frac{\text{ar}(OTQS)}{\text{ar}(OPQRO)}=\frac{\int_0^1\sqrt{x}\,dx-\int_0^1 x^2\,dx}{1\times1}=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

$$\because\ P(A)+P(B)=\frac{2}{3}+\frac{2}{3}\ne 1$$

So, A and B are not exhaustive.

$$P(A)\cdot P(B)=\frac{2}{3}\cdot\frac{2}{3}=\frac{4}{9}\ne P(A\cap B)$$

So, A and B are not independent.

$$P(A\cup B)=1,\ P(A)+P(B)=\frac{2}{3}+\frac{2}{3}\ne P(A\cup B)$$

So, A and B are not mutually exclusive.

12 Total number of ways of selecting 2 different numbers from $\{0, 1, 2, \ldots, 10\} = {}^{11}C_2 = 55$

Let two numbers selected be x and y.

Then, $\quad x + y = 4m \quad \ldots$(i)

and $\quad x - y = 4n \quad \ldots$(ii)

$\Rightarrow\ 2x = 4(m+n)$ and $2y = 4(m-n)$

$\Rightarrow\ x = 2(m+n)$ and $y = 2(m-n)$

Thus, x and y both are even numbers.

x	y
0	4, 8
2	6, 10
4	0, 8
6	2, 10
8	0, 4
10	2, 6

$\therefore$ Required probability = $\frac{6}{55}$

13 Let E = event when each American man is seated adjacent to his wife and A = event when Indian man is seated adjacent of his wife.

Now, $n(A\cap E) = (4!)\times(2!)^5$

$\because$ While grouping each couple, we get 5 groups which can be arranged in $(5-1)!$ ways, and each of the group can be arranged in 2! ways.

and $\quad n(E) = (5!)\times(2!)^4$

$\because$ While grouping each American man with his wife, we get 4 groups. These 4 groups together with indian man and his wife (total 6) can be arranged in (6–1)! ways and each of the group can be arranged in 2! ways.

$$\therefore\ P\left(\frac{A}{E}\right)=\frac{n(A\cap E)}{n(E)}=\frac{(4!)\times(2!)^5}{(5!)\times(2!)^4}=\frac{2}{5}$$

14 We observe that $7^1, 7^2, 7^3$ and 7^4 ends in 7, 9, 3 and 1 respectively. Thus, 7^l ends in 7, 9, 3 or 1 according as l is of the form $4k+1$, $4k+2$, $4k-1$ or $4k$, respectively. If S is the sample space, then $n(S) = (100)^2$. $7^m + 7^n$ is divisible by 5 if (i) m is of the form $4k+1$ and n is of the form $4k-1$ or (ii) m is of the form $4k+2$ and n is of the form $4k$ or (iii) m is of the form $4k-1$ and n is of the form $4k+1$ or (iv) m is of the form $4k$ and n is of the form $4k+1$.

Thus, number of favourable ordered pairs $(m, n) = 4\times25\times25$.

Hence, required probability is $\frac{1}{4}$.